

Methods and limitations of NMR data inversion for fluid typing

Boqin Sun and Keh-Jim Dunn*

ChevronTexaco Energy Technology Company, San Ramon, CA 94583, USA

Received 5 November 2003; revised 25 March 2004

Available online 6 May 2004

Abstract

We introduce two NMR inversion methods within the framework of 1D NMR to extract fluid saturations by varying echo spacing and wait time. The first method connects the T_2 distribution of each fluid with the overall apparent T_2 distribution using a shift matrix. Each fluid's saturation and T_2 distribution are extracted by minimizing the difference between the model T_2 distributions and measured apparent T_2 distributions. The second method relates a model T_2 distribution of each fluid with CPMG echo trains using a global evolution matrix that governs the evolution of magnetization under T_1 , T_2 relaxation, and diffusion. These methods will be useful whenever data are not sufficient for 2D NMR inversion. They are also much faster than 2D for fluid typing. We also point out an inherent limitation associated with NMR inversion methods for fluid typing. Whenever there is singularity in the inversion matrix caused by similar behavior of model function for different fluids, most inversion algorithms remove the solution space associated with the singularity and choose a solution vector of the minimum length. This results in equal proportions of different fluids in the final answer. If prior knowledge such as saturation or T_2 shape of the oil is available, there are several methods to tailor the solution to our desired outcome. However, if there is no prior knowledge available, such ambiguity always exists irregardless of the inversion schemes. © 2004 Elsevier Inc. All rights reserved.

Keywords: NMR relaxation; Inversion; NMR logs; Fluid typing

1. Introduction

Nuclear magnetic resonance logging technology has been widely used in the petroleum industry to measure porosity and permeability in subsurface formation. It typically uses a permanent magnet with a proton Larmor frequency around several hundred kHz to a few MHz and CPMG pulse sequences to measure T_2 distribution of the formation fluids [1]. Since the formation fluids, such as water, oil, and gas, have widely different diffusion coefficients and moderately different relaxation times, it is possible to differentiate these fluids using the diffusion effect in a magnetic field gradient with different echo spacings and/or different degrees of polarization recoveries with different wait times.

There were several NMR logging methods proposed for hydrocarbon typing in the past [2–5]. These methods basically use either different echo spacings or different wait times to create differences in T_2 distribution for

hydrocarbon typing. They were primarily methods within the 1D NMR framework. Recent advances have extended to 2D NMR inversions [6–12], some of which utilize two-window type modified CPMG sequences [6–9], and another [12] simply uses regular CPMG sequences.

However, 2D inversion require multiple echo trains (4–6 or more) with different echo spacings and wait times. This kind of data is not always available. Common NMR log data are still handled with 1D inversion scheme. Alternative inversion methods within the 1D framework still merit discussion and dissemination. Here, we introduce two 1D methods which are similar in principle to the previous time domain approaches, but the mathematical manipulation is different where a shift matrix scheme in the T_2 domain is used to incorporate the diffusion effect. The first method is “Fluid typing by Editing T_2 distributions” (FET). It connects the T_2 distribution of each fluid with the overall T_2 distribution using a shift matrix. Each fluid's saturation and T_2 distribution are extracted by minimizing the difference between the model T_2 distributions and measured apparent

* Corresponding author. Fax: 1-925-842-3442.

E-mail address: kedu@chevrontexaco.com (K.-J. Dunn).

T_2 distributions. This method allows solving for fluid saturations even in T_2 domain. The second method, “Global Inversion of Fluid Typing” (GIFT), is an extension of the first method. It couples the T_2 distribution of each fluid with the CPMG echo trains directly through a global evolution matrix and solve the problem in time domain.

We also point out an inherent limitation associated with the NMR inversion methods for fluid typing, whenever multiple fluids exist in a T_2 region which is insensitive to the diffusion effect. This limitation exists both for 1D and 2D inversion. Whenever this occurs, it results in singularity in the inversion matrix. Most algorithms choose a minimum solution vector as a solution. This leads to equal proportions of the fluids in the T_2 region which is insensitive to the diffusion effect. Thus, it becomes apparent that many of the practices which address this problem are somewhat arbitrary.

2. Fluid typing by editing T_2 distributions

We now introduce the first method, “Fluid typing by Editing T_2 distributions” (FET). In a NMR logging job, we typically acquire CPMG echo trains. The decay of the echo train can be characterized by a multi-exponential decay function as follows:

$$b_i = \sum_{j=1}^{N_R} A_j e^{-t_i/T_j} + \varepsilon_i = \sum_{j=1}^{N_R} A_j E_{ij} + \varepsilon_i, \quad i = 1, \dots, N_E, \quad (1)$$

where $E_{ij} \equiv e^{-t_i/T_j}$, b_i is the i th echo amplitude, $t_i = it_E$ is the decay time, t_E is the time between echoes, T_j is a set of N_R pre-selected relaxation times equally spaced on a logarithmic scale, N_E is the number of echoes, and ε_i is the noise of the i th echo. A_j is the T_2 amplitude associated with the relaxation time T_j to be solved by the model function we assume in Eq. (1).

Here, we assume that t_E is small enough that any diffusion effect is negligible. We also assume that the wait time between the CPMG excitations is sufficient that the polarization factor is close to 1 and can be left out in Eq. (1). The polarization factor associated with A_j is $1 - \exp(-WT/rT_j)$, where WT is the wait time and r is the T_1/T_2 ratio. Whenever the wait time is not sufficient, we need to put the factor back in the model for proper consideration.

Using matrix notation, Eq. (1) can be written as

$$B = EA, \quad (2)$$

where $B = [b_1, b_2, \dots, b_{N_E}]^T$ is the data vector representing the whole echo train, $E = [E_{ij}]$ is the evolution matrix of the magnetization under relaxation, and $A = [A_1, A_2, \dots, A_{N_R}]^T$ is the solution vector representing the amplitude of the T_2 distribution. We have used the

superscript “T,” which stands for *Transpose*, to indicate that both B and A are column vectors. Eqs. (1) or (2) can be solved by any least squares algorithm subject to the non-negativity constraint for A_j . Two commonly used methods are singular value decomposition (SVD) [13] and Butler–Reeds–Dawson scheme (BRD) [14]. Sometimes, their combination is used [6].

Now let us suppose that we acquire a set of CPMG echo trains with different echo spacings in a magnetic field gradient. Due to diffusion effect, the apparent T_2 of a pore fluid is given by

$$\frac{1}{T_{2a}} = \frac{1}{T_{2b}} + \frac{1}{T_{2s}} + \frac{1}{T_{2D}}, \quad \frac{1}{T_{2s}} = \rho_2 \frac{S}{V}, \quad \frac{1}{T_{2D}} = \frac{1}{12} D(\gamma g t_E)^2, \quad (3)$$

where T_{2a} is the apparent T_2 , T_{2b} is the bulk T_2 relaxation time, T_{2s} is the surface relaxation time (ρ_2 -surface relaxivity, S -pore surface, and V -pore volume), T_{2D} is the equivalent relaxation time caused by the self-diffusion of fluid molecules in a magnetic field gradient g , D is the diffusion coefficient, and γ is the gyromagnetic ratio.

When t_E is very small, the term associated with the diffusion effect can be neglected. The apparent T_2 can be approximated by T_{2s} because $T_{2b} \gg T_{2s}$. However, when t_E is large and the diffusion effect can no longer be neglected, the apparent T_2 will reduce. Such reduction results in a shift of the T_2 distribution towards shorter relaxation times. The shift is larger for long and less for short relaxation times. It is also larger for pore fluids with larger diffusion coefficients. The latter effect allows us to differentiate pore fluids based on the contrast of their diffusion coefficients.

To model this problem, we assume that the individual T_2 distributions for water, oil, and gas at $t_E = 0$ are given by

$$a^F = [a_1^F, a_2^F, \dots, a_{N_F}^F]^T, \quad (4)$$

where the superscript $F = (\text{water, oil, gas, } \dots) = (1, 2, 3, \dots)$, N_F is the number of pre-selected T_2 components for the T_2 distribution of fluid F . Note that a^F is defined at $t_E = 0$, and hence does not contain any diffusion effect. The total T_2 distribution for all pore fluids at $t_E = 0$ is given by

$$A_j^0 = \sum_{F=1}^3 a_j^F. \quad (5)$$

When $t_E \neq 0$, the inverted T_2 distribution using the model function in Eq. (1) contains diffusion effect, and the spectrum tends to shift to shorter relaxation times when compared with A_j^0 . We denote this T_2 as the apparent T_2 distribution

$$A^M = [A_1^M, A_2^M, \dots, A_{N_M}^M]^T, \quad (6)$$

where M indicates the M th echo train with the echo spacing t_E^M .

In general, N_F , N_M , and N_R can be different. However, for simplicity, we assume $N_F = N_M = N_R$. If we choose a large N_R , then it is approximately true we can shift the amplitude at T_j for $t_E = 0$ to a lower index of shorter relaxation time for $t_E \neq 0$. The amount of such a shift depends on the magnitude of T_j , and is given by

$$m_j^F = N_R \frac{\log T_{2j} + \log \left(\frac{1}{T_{2j}} + \frac{1}{12} D^F (\gamma g t_E)^2 \right)}{\log T_{2\max} - \log T_{2\min}}, \quad (7)$$

where $j = (1, 2, \dots, N_R)$, $F = (\text{water, oil, gas, } \dots)$, $T_{2\max}$ and $T_{2\min}$ define the range of the T_2 distribution of each fluid, and D^F is the diffusion coefficient of the fluid F . The diffusion coefficients for water and gas are usually known for a given temperature and pressure. Thus, the amount of shift for water and gas can be easily computed. Since the diffusion coefficient for oil is not known beforehand and that it usually has a distribution of values, we adopt the model suggested by Freedman et al. [5] to express the distribution of diffusion coefficients for oil D_j in terms of the distribution of relaxation times for oil T_j as

$$D_j = \frac{b}{a} T_j. \quad (8)$$

The constants a and b are from the respective relationships of T_2 relaxation times and diffusion coefficient with respect to viscosity η and absolute temperature T_K , i.e.,

$$T_{2,\text{LM}} = \frac{a T_K}{\eta}, \quad D_{\text{LM}} = \frac{b T_K}{\eta}, \quad (9)$$

where ‘‘LM’’ means the logarithmic average over the respective distributions, and T_K is in Kelvin.

Then the amplitude at T_{2j} for the apparent T_2 distribution inverted from the M th echo train of echo spacing t_E^M is given by

$$A_j^M = \sum_{F=1}^3 a_{j+m_j^F}^F + \sigma_j, \quad j = 1, 2, \dots, N_R, \quad (10)$$

which indicates the apparent amplitude at index j was shifted from a higher index $j + m_j^F$ of each fluid. Here, σ_j represents the misfit between the model and the inverted apparent T_2 amplitude A_j^M . We now introduce a shift matrix S^M for such an operation

$$A_j^M = \sum_{F=1}^3 \sum_{k=1}^{N_R} S_{j,(F-1)N_R+k}^M a_k^F + \sigma_j, \quad j = 1, 2, \dots, N_R, \quad (11)$$

where

$$S_{j,(F-1)N_R+k}^M = \begin{cases} 1 & \text{for } k = \text{NINT}(j + m_j^F), \\ 0 & \text{for } k \neq \text{NINT}(j + m_j^F) \end{cases} \quad (12)$$

and $F = (1, 2, 3, \dots)$. NINT means rounding to the nearest integer. Thus, each row of S^M has only three non-zero elements, one for each F . Practically, m_j^F is almost never an integer. This means that the shifted T_2

will not be located at one of our pre-selected T_2 relaxation times. To be more general, we can redistribute the shifted T_2 amplitude to its nearest pre-selected relaxation times using a Gaussian-type function. This re-proportion of the T_2 amplitude is symmetrical about the shifted T_2 relaxation time and has the shape of a Gaussian distribution. The sum of the re-proportioned amplitudes should be equal to the original T_2 amplitude at $t_E = 0$. With this Gaussian distribution for the shifted T_2 amplitude, the shift matrix now has the following form:

$$S_{j,(F-1)N_R+k}^M = c \exp \left[-4 \left(\frac{k - j - m_j^F}{N + 1} \right)^2 \log 2 \right] \quad (13)$$

for $k = \text{NINT}(j + m_j^F - N), \dots, \text{NINT}(j + m_j^F + N)$, and

$$S_{j,(F-1)N_R+k}^M = 0 \quad (14)$$

for $k < \text{NINT}(j + m_j^F - N)$, and $k > \text{NINT}(j + m_j^F + N)$, where we choose the normalizing constant c to ensure the sum of the weight of the shifted indices is equal to 1, i.e.,

$$\sum_k S_{j,(F-1)N_R+k}^M = 1. \quad (15)$$

Here, $N + 1$ defines the half width of the Gaussian distribution.

We now concatenate all inverted M apparent T_2 distributions, shift matrices, and the individual T_2 distributions of pore fluids at $t_E = 0$ and define the following:

$$A_C \equiv \begin{bmatrix} A^1 \\ \vdots \\ A^M \end{bmatrix}, \quad E_S \equiv \begin{bmatrix} S^1 \\ \vdots \\ S^M \end{bmatrix}, \quad a_C \equiv \begin{bmatrix} a^{\text{water}} \\ a^{\text{oil}} \\ a^{\text{gas}} \end{bmatrix}, \quad (16)$$

where A_C is a column vector of $M \times N_R$ elements, E_S is a matrix of $(M \times N_R) \times 3N_R$, each shift matrix S^M has a dimension of $N_R \times 3N_R$, and a_C is a column vector of $3N_R$ elements.

The problem can be formulated in matrix form as

$$A_C = E_S a_C. \quad (17)$$

Here, the shift matrix E_S contains diffusion effects due to finite t_E in a field gradient, A_C is the apparent T_2 distributions from various t_E and is used as input data, and a_C is the solution which we wish to determine. Eq. (17) can be solved by any standard least squares algorithms subject to the non-negativity constraint of a^F .

Earlier methods for hydrocarbon typing [2] such as the shift and differential spectrum methods, often run into situations where subtraction of two apparent T_2 distributions resulted in negative amplitudes. Consequently, many analyses were done in time rather than T_2 domain to bypass the uncertainty of apparent T_2 distributions due to inversion error. We show here that the FET can be an effective method for fluid typing with direct inversion from T_2 distributions without using the time domain data, and that fluid typing need not be done by the time domain analysis only.

3. Global inversion for fluid typing

The second method, “Global Inversion for Fluid Typing” (GIFT), for obtaining water, oil, and gas saturations is an improvement over the first method. As one might expect that in the first method, the error of measurements and the error of inversion to apparent T_2 distributions are compounded in the two-step process. It is natural to think of combining the two steps into one, then the error of the solution will be determined solely by the error of measurements, and GIFT is such one step process.

We note that each apparent T_2 distribution A^M is obtained in the following way:

$$B^M = E^M A^M, \quad (18)$$

where $E_{ij}^M = (1 - e^{-WT/rT_j})e^{-t_i/T_j}$ and $B^M = [b_1, b_2, \dots, b_{N_E}]^T$ is the echo train with echo spacing t_E^M . To reduce Eqs. (17) and (18) into one step, we define the following:

$$B_C \equiv \begin{bmatrix} B^1 \\ \vdots \\ B^M \end{bmatrix}, \quad E_C \equiv \begin{bmatrix} E^1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & E^M \end{bmatrix}, \quad (19)$$

where B_C is a big column vector of length $n = \sum_{i=1}^M N_{E_i}$ obtained by concatenating echo trains of all M measurements of M different t_E 's or wait times or combination thereof, and E_C is a compound matrix of size $n \times (M \times N_R)$ by placing all matrices along the diagonal. Thus, we have

$$B_C = E_C A_C = E_C E_S a_C = E_G a_C, \quad (20)$$

where we define the global evolution matrix $E_G \equiv E_C E_S$. Now, the T_2 distributions of pore fluids at $t_E = 0$, a_C , are coupled directly to the data vector B_C through the matrix E_G . They can be solved by Eq. (20).

4. Comparison with other approaches

A more traditional approach for NMR inversion for fluid typing is a direct inversion without the use of the shift matrix, such as the one described by Freedman et al. [5], i.e.,

$$\begin{aligned} b_i^M &= \sum_F \sum_{j=1}^{N_R^F} a_j^F (1 - e^{-WT_M/T_{1,j}}) e^{-t_i/T_{2D^F}} e^{-t_i/T_{2,j}} + \epsilon_i^M \\ &= \sum_F \sum_{j=1}^{N_R^F} a_j^F E_{ij}^{MR^F} + \epsilon_i^M, \quad i = 1, \dots, N_E^M, \end{aligned} \quad (21)$$

where b_i^M is the i th echo of the M th echo train obtained with the echo spacing t_E^M , and a_j^F is the solution of the T_2 amplitudes we wish to determine for the pore fluid F . For comparison purpose, we shall call this traditional scheme “MRF” method.

Comparing the GIFT with MRF, we note that the diffusion effect indicated by $e^{-t_i/T_{2D^F}}$ in MRF has been completely replaced by the shift matrix S^M in GIFT. The regularization of solution due to finite noise level in MRF has been partially replaced by the Gaussian shaped re-proportion of the shifted T_2 in GIFT. Here, we say “partially” because the application of SVD for matrix inversion in GIFT contains certain level of regularization as well.

In fact, there are several factors affecting the construction of the shift matrix. The Gaussian spread for the shifted T_2 is the first level consideration coming from the regularization for noise smoothing. Based on this consideration alone, the Gaussian spread for a shift from a short T_2 should be larger than that from a long T_2 , as the short T_2 component is usually more noisy due to less number of echoes. However, we believe such subtle differences may be smeared due to more serious factors considered below.

When the logging tool in the probed region has a distribution of field gradients, it has to be properly considered in the shift matrix. If the distribution is described by the following:

$$\int_0^\infty f(g) dg = 1, \quad (22)$$

where $f(g)$ is the volume fraction that has a gradient g , then the whole distribution needs to be applied to each relaxation time. Hence, the shifted T_2 for each relaxation time will be a superposition of many Gaussian spreads, each corresponds to a single gradient g with an amplitude proportional to $f(g)$. The integration of the final form, which most likely will not be Gaussian, should be equal to the original unshifted T_2 amplitude.

Our experience has indicated that a reasonably good result is obtained if we choose a width for the Gaussian spread using $N = 2$ in Eq. (15). The actual comparison between MRF and GIFT is discussed in later sections.

5. Inherent limitation for fluid typing

5.1. Singularity of the inversion matrix

It has long been observed that whenever there is a T_2 region, such as the regime for irreducible water saturation or heavy oil, where the diffusion effect is not large enough to produce discernible contrast between water and oil, the inversion of NMR data invariably produces equal amounts of water and oil for the same T_2 relaxation time. This is because of the singularity or near singularity of the inversion matrix. Since Eq. (1) is an ill-posed problem, the solution is not unique. Most algorithms choose the smallest solution vector as the answer, which in general, resulted in equal proportion of water and oil. In the following, we shall illustrate

this using the singular value decomposition (SVD) method.

Let us consider an extreme case that a T_2 distribution is confined to the region of short T_2 relaxation times where the diffusion effect does not cause T_2 components any significant shift. Suppose we have only water present, and the echo train data we acquired for the water can be described by the following:

$$b_i^w = E_{ij}^w y_j^w, \\ E_{ij}^w = \exp \left[-t_i \left(\frac{1}{T_j^w} + \frac{1}{12} D^w (\gamma g t_E)^2 \right) \right] \approx \exp[-t_i/T_j^w], \quad (23)$$

where we used the superscript “w” to indicate *water*, and $1/T_j^w \gg \frac{1}{12} D^w (\gamma g t_E)^2$. If the data vector is denoted by $B^w = [b_1^w, b_2^w, \dots, b_n^w]^T$, after solving $B^w = E^w Y^w$, we obtain the solution for the T_2 distribution for water as $Y^w = [y_1^w, y_2^w, \dots, y_m^w]^T$.

Separately, if we have a crude oil which also has similar short T_2 relaxation times. The echo train we acquired for the oil can be described by

$$b_i^o = E_{ij}^o y_j^o, \\ E_{ij}^o = \exp \left[-t_i \left(\frac{1}{T_j^o} + \frac{1}{12} D^o (\gamma g t_E)^2 \right) \right] \approx \exp[-t_i/T_j^o], \quad (24)$$

where we used the superscript “o” to indicate *oil*, and $1/T_j^o \gg \frac{1}{12} D^o (\gamma g t_E)^2$. The data vector is denoted by $B^o = [b_1^o, b_2^o, \dots, b_n^o]^T$ which can be very different from B^w . After solving $B^o = E^o Y^o$, we get the solution for the T_2 distribution for oil as $Y^o = [y_1^o, y_2^o, \dots, y_m^o]^T$.

If we now have both water and the crude oil present, and we acquire an echo train of n echoes which contains signals from both water and oil. Not knowing the amount of water or oil, we would set up the problem as follows:

$$b_i = E_{ij} y_j, \quad (25)$$

where

$$E = [E^w \ E^o], \quad \text{and} \quad Y = [y_1^w, \dots, y_m^w, y_1^o, \dots, y_m^o]^T. \quad (26)$$

We have chosen the same set of T_2 relaxation times, i.e., T_1, \dots, T_m , for both the water and oil for the inversion. Thus, E is a $n \times 2m$ matrix with two similar matrices, E^w and E^o , side by side placed together. Because $E_{ij}^w \approx E_{ij}^o$ for all i and j , if E_{ij} is non-singular, and has singular values $\lambda_1, \lambda_2, \dots, \lambda_m$, then E would be singular with a rank of m and a null space of m -dimensions. The singular value decomposition of E resulted in the following:

$$E = U W W^T, \quad (27)$$

where $U = [U_{ij}]$, $V = [V_{ij}]$,

$$\sum_{k=1}^n U_{ik} U_{jk} = \delta_{ij}, \quad 1 \leq i, j \leq 2m, \quad (28)$$

$$\sum_{k=1}^{2m} V_{ik} V_{jk} = \delta_{ij}, \quad 1 \leq i, j \leq 2m, \quad (29)$$

and W is a diagonal matrix given by

$$\text{dia } W = (\lambda_1, \dots, \lambda_m, 0, \dots, 0). \quad (30)$$

The matrix V has the following form:

$$V = \begin{bmatrix} v_{11}/a & \cdots & v_{1m}/a & V_{1,m+1} & \cdots & V_{1,2m} \\ \vdots & & \vdots & \vdots & & \vdots \\ v_{m1}/a & \cdots & v_{mm}/a & \vdots & & \vdots \\ v_{11}/a & \cdots & v_{1m}/a & \vdots & & \vdots \\ \vdots & & \vdots & \vdots & & \vdots \\ v_{m1}/a & \cdots & v_{mm}/a & V_{2m,m+1} & \cdots & V_{2m,2m} \end{bmatrix} \quad (31)$$

and the factor $a = \sqrt{2}$, where the top left and the bottom left quadrants of the matrix V are occupied by matrices $V^w = [v_{ij}^w]$ and $V^o = [v_{ij}^o]$ (except the factor a) which were obtained from the singular value decomposition of E^w or E^o , respectively. Naturally, this is expected because we have assumed that $E_{ij}^w \approx E_{ij}^o$, hence, $v_{ij}^w \approx v_{ij}^o = v_{ij}$. Thus, the presence of the factor $a = \sqrt{2}$ just helps normalize the vectors to unit vectors. The last m column vectors $V_{i,m+1}$ through $V_{i,2m}$ are irrelevant, as they do not play any role in the inversion.

The pseudoinverse of W , obtained by taking the reciprocal of non-zero singular value and replacing zero singular value with zero, created a null space of m -dimensions which leads to

$$Y = \begin{bmatrix} y_1^w \\ \vdots \\ y_m^w \\ y_1^o \\ \vdots \\ y_m^o \end{bmatrix} = \begin{bmatrix} v_{11}/a & \cdots & v_{1m}/a & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ v_{m1}/a & \cdots & v_{mm}/a & \vdots & & \vdots \\ v_{11}/a & \cdots & v_{1m}/a & \vdots & & \vdots \\ \vdots & & \vdots & \vdots & & \vdots \\ v_{m1}/a & \cdots & v_{mm}/a & 0 & \cdots & 0 \end{bmatrix} \\ = \begin{bmatrix} b'_1/\lambda_1 \\ \vdots \\ b'_m/\lambda_m \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad (32)$$

where $b'_i = U_{ji} b_j$ and we have also replaced the last m column vectors with zeroes as they are associated with the null space and do not play any role in the inversion. Thus, the fact that the top left and the bottom left quadrants of the matrix V are identical to each other leads to the natural result that $y_i^w = y_i^o$, i.e., equal partitioning of water and oil components as the solution. Since $y_i^w + y_i^o = \sum_j v_{ij} b'_j / \lambda_j = s$ is a constant, the requirement that $(y_i^w)^2 + (y_i^o)^2 = (y_i^w)^2 + (s - y_i^w)^2$ be a minimum leads to $y_i^w = y_i^o$. Naturally, $Y + Y'$ for any Y'

in the null space is a solution to the problem as well. Again, the choice of the solution vector having the minimum length leads to the same outcome. For other inversion methods with a norm smoothing regularization term, the requirement of the solution vector being the minimum length leads to the same result.

Such reasoning can be easily extended to any distributions of T_2 relaxation times. As long as there is a region of T_2 which is insensitive to the diffusion effect, the inversion matrix containing the diffusion effect in the model will be singular. If there is no signal from that part of the T_2 region, the inversion would not produce any T_2 components in that region. However, if there is signal from that part of the T_2 region, the inversion will invariably produce equal amounts of water and oil in that T_2 region because of the minimum length requirement for the solution vector. This limitation is not only inherent in one-dimensional NMR inversion, as it is discussed here. It also exists for two-dimensional NMR inversion.

5.2. Reducing the number of relaxation times

Knowing the behavior of the inversion algorithms, one can manipulate the outcome of oil saturation by reducing the number of relaxation times for the oil in the region of T_2 where the ambiguity occurs. As indicated in Fig. 1, we show schematically a hypothetical inversion result where both water and oil have two bumps, one at short relaxation time and the other at long relaxation time. The ones at long relaxation time for both water and oil are most likely real, as the behaviors of E_{ij}^w and E_{ij}^o are very different, whereas the ones at short relaxation time are ambiguous, because $E_{ij}^w \approx E_{ij}^o$ in this region. The equal proportions of the water and oil are a direct result of the inversion which chooses the solution

vector of the minimum length. It is not a reflection of the true water and oil saturations in that T_2 region. As the number of relaxation times for oil in the model is reduced, the equal partitioning of water and oil having the same relaxation times in the short T_2 region is still in effect, whereas the oil amplitudes in the long T_2 region increase to compensate for the effect of the reduction of the number of relaxation times. Therefore, after normalizing the display for oil with the same number of relaxation times as that for water, the oil at long T_2 region maintain the same amplitude as that before the reduction of relaxation times, whereas the oil at short T_2 is effectively suppressed. Obviously, such manipulation is arbitrary. It is only justified when prior knowledge of the T_2 shape of the oil is available through laboratory calibration or other means. If the location of the T_2 relaxation times for the oil is known, and it is not in the T_2 region which is insensitive to diffusion effect, one can simply set up the model by not having any T_2 relaxation times in that region. Or, one can set up the model by having non-overlapping T_2 relaxation times for oil and water. However, all these manipulations require prior knowledge of the T_2 behavior for the oil.

5.3. Applying weighting factor using other logs

Such manipulation can also be accomplished by applying weighting factor using other logs. We introduce a weight matrix, whose element is 1 for the term we wish to retain, but smaller than 1 for the term we wish to suppress. For example, since the presence of oil and gas is usually associated with high resistivity value. We can filter out the short T_2 components in oil and gas distributions using a weighting factor

$$w_R(T_2) = \frac{1 - e^{-T_2/cR}}{1 + e^{-T_2/cR}}, \quad (33)$$

where R is the resistivity of the formation and c is a fitting constant. Thus, for any elements of oil and gas distributions whose $T_2 \ll cR$ will be suppressed. In fact, the suppression for gas is redundant, because the T_2 behavior of gas is very different from that of water and oil. We also know that when the neutron and density logs cross with each other usually indicates the presence of gas. Thus, we can filter out water and oil using a weighting factor

$$w_\rho(T_2) = \frac{e^{-c_0 T_2 (\rho_H - \rho_B - c_1)}}{1 + e^{-c_0 T_2 (\rho_H - \rho_B - c_1)}}, \quad (34)$$

where ρ_H is the neutron density, ρ_B is the gamma ray density, c_0 , and c_1 are two fitting constants. Of course, the detailed form of each weight factor can be different. We can, in general, express the total weight matrix as a product of $w_R(T_2)$ and $w_\rho(T_2)$, that is,

$$w(T_2) = w_R(T_2)w_\rho(T_2). \quad (35)$$

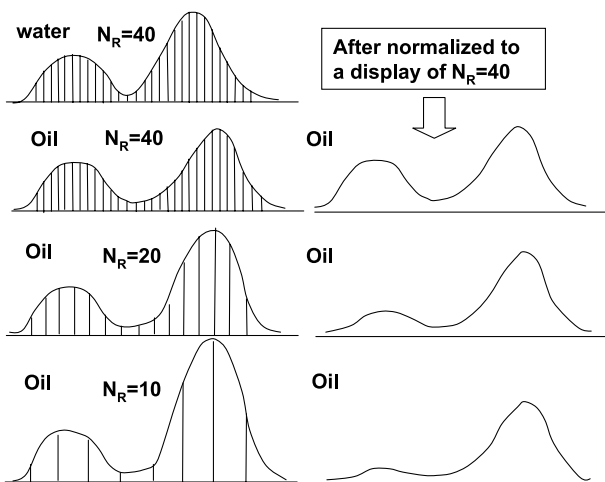


Fig. 1. When the same set of T_2 distribution is used for water and oil, the inversion usually gives equal proportions of water and oil for the short T_2 components. By reducing the number of relaxation times of oil, the oil components at short T_2 can be arbitrarily suppressed.

6. Discussion of results

We have applied the two inversion methods to synthetic data as well as real log data to test their effectiveness. We also analyze the situations where the ambiguity of fluid typing occurs. To test the FET method, we generated a synthetic dataset using equal amount of proton population for water, oil, and gas (an equivalent of 5 porosity units) in a magnetic field gradient of 13.5 G/cm. The diffusion coefficients used for water, oil, and gas are 5.54×10^{-5} , 1.86×10^{-6} , and

Table 1
Echo spacing and apparent T_2 distribution of water, oil, and gas, all in milliseconds

Echo spacing	0	1.2	2.4	3.6	4.8
T_{2a} (water, input)	200	170	118	78	53
T_{2a} (oil, input)	395	389	370	343	311
T_{2a} (gas, input)	2667	76	19	9	5
$T_{2,LM}$ (water, output)	212	162	109	70	47
$T_{2,LM}$ (oil, output)	362	331	321	304	275
$T_{2,LM}$ (gas, output)	1722	66	17	7	4

They were used to generate synthetic CPMG echo trains for FET inversion. The $T_{2,LM}$ obtained from the inversion are shown for comparison.

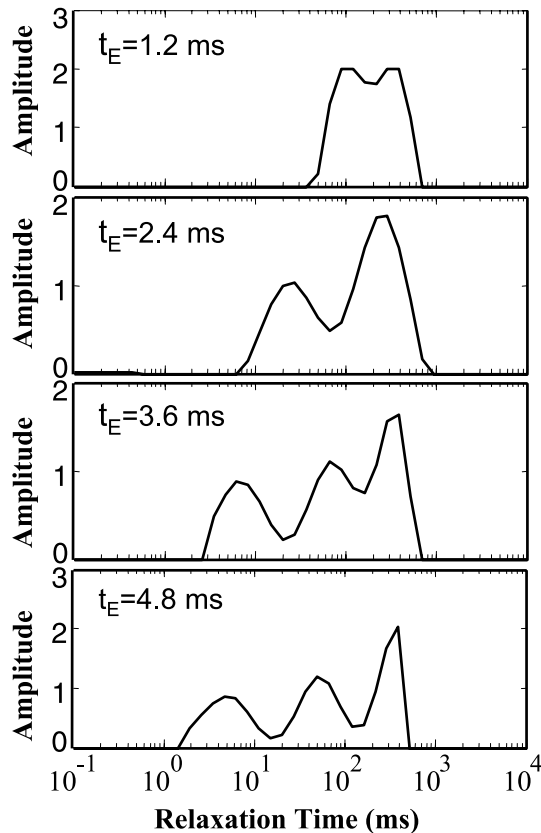


Fig. 2. The apparent T_2 distributions inverted from CPMG echo trains with echo spacings of 1.2, 2.4, 3.6, and 4.8 ms. The echo trains were synthetically generated from water, oil, and gas with their T_2 's listed in Table 1.

$8.1 \times 10^{-4} \text{ cm}^2/\text{s}$, respectively. Table 1 listed different echo spacings and their corresponding apparent T_2 in milliseconds for water, oil, and gas. They were used to generate CPMG echo trains with a 0.3 p.u. white noise. The inverted apparent T_2 distributions for different t_E 's using SVD are shown in Fig. 2. These apparent T_2 distributions were then used as input for the application of FET method. The results of the solution are shown in Fig. 3 as individual T_2 distributions for water, oil, and gas, as well as the sum total at different echo spacings for comparison with those in Fig. 2. The logarithmic average, $T_{2,LM}$, for water, oil, and gas were also listed in Table 1 for comparison with the input values. The agreement is reasonably good.

In the second test using synthetic data, we used the GIFT method for the inversion. Table 2 listed all properties of the input fluids for the model, where we assumed equal amount of proton population (an equivalent of 5 porosity units) for each pore fluid, i.e., oil, oil based mud filtrate (OBMF), water (irreducible water, BVI), water (free fluid index, FFI), and gas. We assume a magnetic field gradient of 13.5 G/cm. The T_1 and T_2 are arbitrary values where we assume a T_1/T_2 ratio of 1.5, and the T_2 values are for $t_E = 0$.

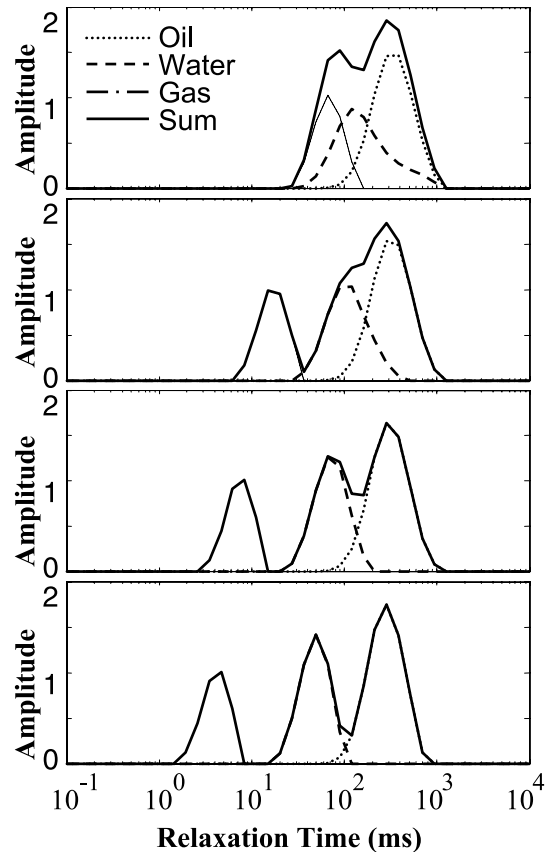


Fig. 3. The individual T_2 distributions for water, oil, and gas inverted from the apparent T_2 distributions in Fig. 2 using FET. The total T_2 distributions for different t_E 's were recomputed using the shift matrix.

Table 2
Properties of input fluids used to generate synthetic CPMG data for testing GIFT inversion method

	Oil	OBFM	Water (BVI)	Water (FFI)	Gas (HI=1)
Saturation (p.u.)	5	5	5	5	5
Diffusion const (10^{-5} cm ² /s)	0.01	0.3	5.3	5.3	80.4
T_{2s} (ms)	143	431	6.6	467	1937
T_1 (ms)	216	647	10	700	2906

Our purpose is to see how accurate and effective we can recover the T_2 values using the GIFT inversion.

If we set up the inversion matrix using the same set of 30 T_2 relaxation times, equally spaced on a logarithmic scale from 0.1 to 10^4 ms, for water, oil, gas, and OBFM. We immediately realize the inherent problem of fluid typing associated with the singularity of the inversion matrix. The inverted results using GIFT are shown in Fig. 4. We note that in the T_2 region between 0.1 and 20 ms, the T_2 distributions for water, oil, and OBFM, except for gas, are essentially the same. This equal proportions of water, oil, and OBFM, as we discussed earlier, is due to the singularity in the inversion matrix

associated with the similar behaviors of T_2 for these pore fluids in that T_2 region. This singularity leads to equal proportions of water, oil, and OBFM in that region, even though we knew that there should not be any oil or OBFM component there.

If we use a weight matrix, such as Eq. (33), to suppress the contributions for oil and OBFM, we would get the result as shown in Fig. 5. In this example, we used $R = 100 \Omega m$ and $c = 1$. In actual incorporation of log values from resistivity, the value of R can be properly selected, and its sensitivity to the T_2 cutoff value, below which the elements are suppressed, can be properly tailored by choosing appropriate functional form.

We now discuss our last example for a synthetic dataset which fully reveal the inherent problems of NMR inversion for fluid typing associated with the singularity of the inversion matrix. We have four different scenarios, bound water with heavy oil (BWHO), bound water with light oil (BWLO), free water with light oil (FWLO), and free water with heavy oil (FWHO). The T_2 distributions for these four models are shown Fig. 6. The input properties for generating the synthetic CPMG echo trains are listed in Table 3.

To compare the effectiveness of GIFT versus other methods such as MRF for the example shown in Table 3

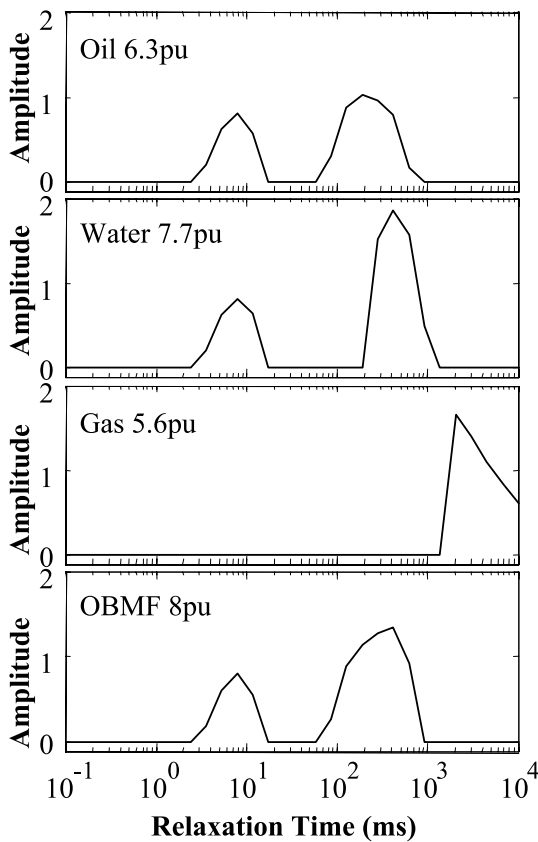


Fig. 4. The inverted T_2 distributions for oil, water, gas, and OBFM showing the equal proportions of oil, water, and OBFM in the short T_2 components.

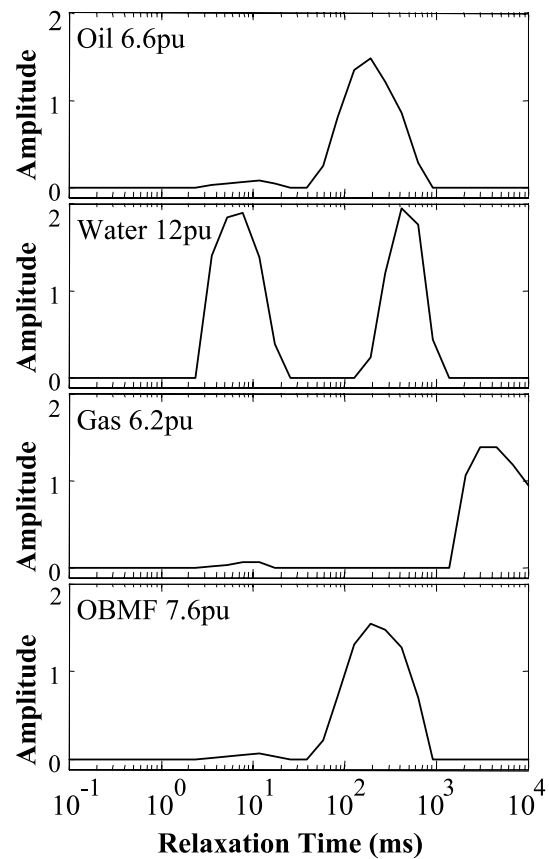


Fig. 5. The inverted T_2 distributions for oil, water, gas, and OBFM, after applying the resistivity filter in the inversion process in Fig. 4.

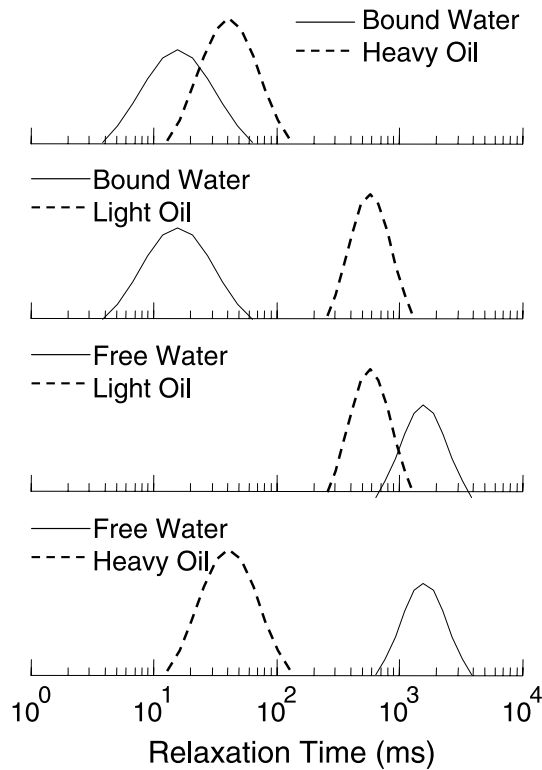


Fig. 6. The T_2 distributions for the four models analyzed for inversion effectiveness, bound water with heavy oil (BWHO), bound water with light oil (BWLO), free water with light oil (FWLO), and free water with heavy oil (FWHO).

Table 3

Properties of input fluids used to generate synthetic CPMG data for the analysis of comparing various inversion methods for fluid typing

	BWHO	BWLO	FWLO	FWHO
ϕ_{water} (p.u.)	15	15	15	15
ϕ_{oil} (p.u.)	20	20	20	20
$T_{2G,\text{oil}}$ (ms)	32	447	447	32
$T_{2G,\text{water}}$ (ms)	12.3	12.3	1225	1225

and Fig. 6, we perform inversions using both GIFT and MRF. We considered three cases: (1) using same set of T_2 relaxation times for water and oil, (2) using a reduced number of T_2 relaxation times for oil, and (3) using different sets of T_2 relaxation times for water and oil. The results for both GIFT and MRF are quite similar to each other. They have the same pitfalls where solutions show equal proportions of water and oil in the short T_2 region. They also have the same response to different types of manipulations of the inversion model functions. Only the results for case (3) are shown in Fig. 7. Again, it underlines the necessity of prior knowledge of the T_2 shape for the oil within the 1D inversion framework. Some of this ambiguity in 1D can be resolved in 2D inversion when the inversion matrix elements E_{ij} are very different for different pore fluids due to large contrast in diffusion coefficients. However, even in 2D, if

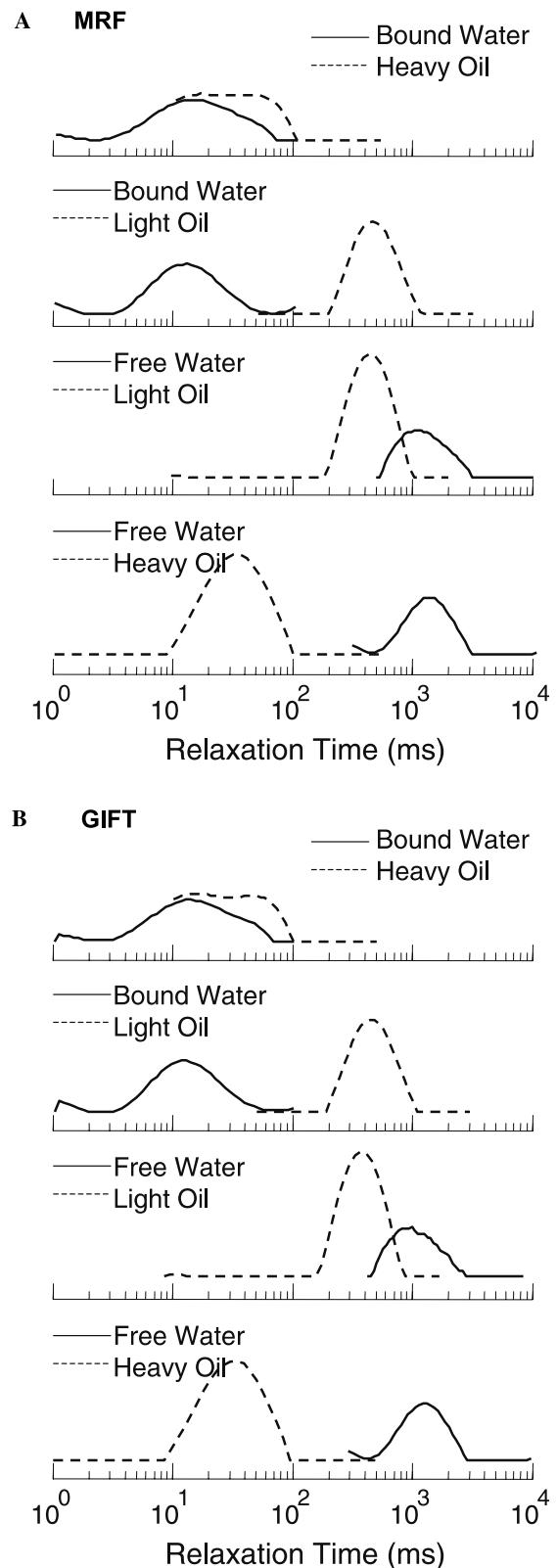


Fig. 7. The inversion results by the traditional method, MRF (A), versus the present method, GIFT (B), using the different sets of T_2 relaxation times for water and oil for the four models discussed in Fig. 6.

singularity exists to due similar elements, there is always ambiguity.

We have also applied MRF and GIFT methods to real NMR stationary logs obtained with four different echo spacings. The raw data were processed by GIFT and MRF. Both methods yield very similar results as shown in Fig. 8. We calculated the oil saturation to be 60%, consistent with the estimate from resistivity log. Based on the T_2 distribution, we also determined the oil viscosity to be about 2.3 cp. Using the relationship between oil viscosity and API, we estimated the API to be

around 24–25, which is very close to the value obtained by the PVT analysis.

7. Conclusions

We introduced two alternative 1D NMR inversion methods that allow us to extract fluid saturations by analyzing the variation of T_2 distribution as a function of echo spacing and wait time. The first method, “Fluid typing by Editing T_2 distributions” (FET), inverts the T_2 distribution of each fluid from the apparent T_2 distributions of different echo spacings using a shift matrix. It offers a workable method in the T_2 domain. The second method, “Global Inversion for Fluid Typing” (GIFT), couples the T_2 distribution of each fluid directly with CPMG echo trains through a global matrix E_G and operates in the time domain. Both methods face the same issue as any other methods for this kind of Laplace transform that the solution is non-unique. An a priori assumption that a large oscillatory behavior in the solution is non-physical dictates the form of the solution. To this end, our methods achieve this through the construction of the shift matrix.

We also delineate the inherent limitation of NMR fluid typing for short T_2 relaxation times associated with the similar T_2 behavior for water and oil. This similarity creates singularity for the inversion matrix, and the inversion invariably produces equal proportions of water and oil, irregardless of the inversion methods. Prior knowledge of the T_2 shape for oil can help resolve the ambiguity either by selecting different T_2 relaxation times for water and oil or using a weight matrix to suppress unwanted components in the short T_2 regime.

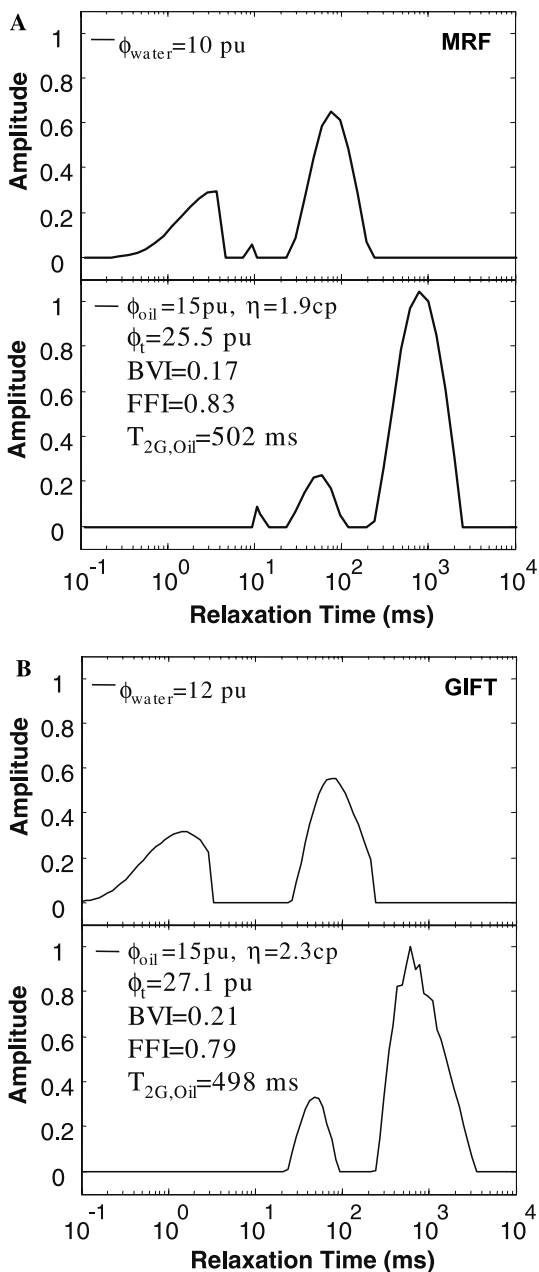


Fig. 8. The inversion results by the traditional method, MRF (A), versus the present method, GIFT (B), for a stationary log obtained with four different t_E 's.

Acknowledgments

We acknowledge the support and encouragement of ChevronTexaco management and their permission to publish the paper. We also thank Kun Hua Chen for providing logging data and Bruce Bilodeau and Don Seeburger for useful discussion on using resistivity as weight factors in NMR inversion

References

- [1] K.J. Dunn, D.J. Bergman, G.A. LaTorraca, Nuclear Magnetic Resonance: Petrophysical and Logging Applications, Pergamon Press, Oxford, 2002.
- [2] R. Akkurt, H.J. Vinegar, P.N. Tutunjian, A.J. Guillory, NMR logging of natural gas reservoir, Log Analyst 37 (1996) 33.
- [3] R. Akkurt, D.M. Marshall, R.Y. Eyvazzadeh, J.S. Gardner, D. Mardon, K.J. Dunn, Determination of residual oil saturation using enhanced diffusion, SPE Paper 49014, 1998.

- [4] S. Chen, D.T. Georgi, O. Olima, H. Gamin, J. Minetto, Estimation of hydrocarbon viscosity with multiple t_E , dual t_W logs, *SPE Reservoir Evaluat. Eng.* 3 (6) (2000).
- [5] R. Freedman, A. Sezginer, M. Flaum, A. Matteson, S. Lo, G.J. Hirasaki, A new NMR method of fluid characterization in reservoir rocks: experimental confirmation and simulation results, *SPE Paper 63214*, Dallas, TX, 2000.
- [6] M.D. Hürlimann, L. Venkataramanan, Quantitative measurement of two dimensional distribution functions of diffusion and relaxation in grossly inhomogeneous fields, *J. Magn. Reson.* 157 (2002) 31–42.
- [7] M.D. Hürlimann, L. Venkataramanan, C. Flaum, The diffusion-spin relaxation time distribution function as an experimental probe to characterize fluid mixtures in porous media, *J. Chem. Phys.* 117 (2002) 10223.
- [8] R. Freedman, N. Heaton, M. Flaum, G.J. Hirasaki, C. Flaum, M.D. Hürlimann, Wettability, saturation, and viscosity using the magnetic resonance fluid characterization method and new diffusion-editing pulse sequences, *SPE Paper 77397*, San Antonio, TX, 2002.
- [9] B. Sun, K.-J. Dunn, Probing the internal field gradient of porous media, *Phys. Rev. E* 65 (2002) 051309.
- [10] B. Sun, K.-J. Dunn, Core analysis with two dimensional NMR, in: *Proceeding of the Society of Core Analysts*, Paper 38, Monterey, CA, September, 2002.
- [11] B. Sun, K.-J. Dunn, J. Lopes, S. Stonard, TIMAS 2D NMR technique provides valuable information for produced oil, Paper EEE presented in 44th Annual Symposium of the Society of Professional Well Log Analysts, Galveston, TX, June 22–25, 2003.
- [12] B. Sun, K.-J. Dunn, B. Bilodeau, S.C. Van Dalen, S. Stonard, M.A., Al-Rushaid, Two dimensional NMR logging and field test results, Paper presented in 45th Annual Symposium of the Society of Professional Well Log Analysts, Noordwijk, The Netherlands, 2004.
- [13] W.H. Press, B.P. Flannery, S.A. Teukolsky, W.T. Vetterling, *Numerical Recipes*, Cambridge University Press, Cambridge, 1986.
- [14] J.P. Butler, J.A. Reeds, S.V. Dawson, Estimating solutions of the first kind integral equations with nonnegative constraints and optimal smoothing, *SIAM J. Numer. Anal.* 18 (1981) 381.